The multiplier Approach based on the SAM as a suitable framework to catch the generation process of inequality in the households income distribution

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For the methodology and a more complete presentation see the original paper: Marisa Civardi, Renata Targetti Lenti, The Sam as a framework to catch the generation process of inequality in the households income distribution, Rivista internazionale di scienze sociali, 2019, n. 4, pp. 327-358.

Aims of the paper are two:

1) to introduce the “global multiplier matrix” **M**, derived from a Social Accounting Matrix (SAM), as a “structural” measure of inequality in the personal income distribution. The values of the global multipliers can be obtained from a SAM considered as a linear model.
2) to enlighten the linkages between the functional and the personal income distribution.

Some numerical examples, referred to the Italian economic system will allow to quantify the effects on the inequality in the personal income distribution of alternative redistributive policies.

The SAM as an accounting framework aimed at linking the primary distribution of income to the functional one

The entire circular flow of income from ***its generation, its distribution*** and finally ***its expenditure*** is captured by the ***Social Accounting Matrix*** (**SAM**).

Each transaction or account has its own row and column. The payments (expenditures) are listed in columns and the receipts are recorded in rows. Row sums must equal the column sums of the corresponding account.

The SAM (Table 1) can be considered as an extension of the traditional input-output framework. It adds some accounts, not included in the Leontief schema, which allow taking in account explicitly the relationships between the ***factorial distribution of income***, the ***primary income distribution to the Institutions*** and the ***final demand.***

The introduction of accounts referred to the Institutions (Households, Private Companies, Government, Rest of the World) allows capturing the link between ***Factors of production*** and the ***Institutions*** which own the different factors of production. The ***secondary distribution of income*** (***disposable income***) is also introduced as the result of transfers between different Institutions, mainly between private Institutions and the

For a complete exposition, see

Table 1. A simplified SAM

Expenditures / Entries	*Activity*	*Factors*	*Institutions*			*Saving/ Invest.*	*Indirect Taxes*	*Rest of the World*	*Total*
			Household	*Companies*	*Government*				
Activity	$T_{1,1}$	0	$T_{1,3}$	$T_{1,4}$	$T_{1,5}$	$T_{1,6}$	0	$T_{1,8}$	y_1
Factors	$T_{2,1}$	0	0	0	0	0	0	$T_{2,8}$	y_2
Household	0	$T_{3,2}$	$T_{3,3}$	$T_{3,4}$	$T_{3,5}$	0	0	$T_{3,8}$	y_3
Companies	0	$T_{4,2}$	$T_{4,3}$	$T_{4,4}$	$T_{4,5}$	0	0	$T_{4,8}$	y_4
Government	0	$T_{5,2}$	$T_{5,3}$	$T_{5,4}$	$T_{5,5}$	0	$T_{5,7}$	$T_{5,8}$	y_5
Saving/ Invest.	0	0	$T_{6,3}$	$T_{6,4}$	$T_{6,5}$	0	0	$T_{6,8}$	y_6
Indirect Taxes	$T_{7,1}$	0	$T_{7,3}$	$T_{7,4}$	$T_{7,5}$	0	0	$T_{7,8}$	y_7
Rest of the World	$T_{8,1}$	$T_{8,2}$	$T_{8,3}$	$T_{8,4}$	$T_{8,5}$	$T_{8,6}$	0	0	y_8
Total	y'_1	y'_2	y'_3	y'_4	y'_5	y'_6	y'_7	y'_8	

The passage from the factorial distribution of income to the personal distribution of income depends on the factors' ownership by the different household groups. The level of total income earned by each household is the result of the translation of the personal endowments (human and physical capital) in earnings.

The earned market incomes of all households can be presented in a block matrix **D** (Table 2)**.**The income received by each of the *H* household group from the factors of production can be easily obtained, pre-multiplying each block of the matrix **D** by a unit transposed row vector $\mathbf{e'}_{nh}$, obtaining the matrix $\mathbf{T}_{3,2}$ (see the SAM in Table 1).

Table 2-Block matrix *D*

Groups of Households		*Categories of Income from Factors*				
1	1	y^1_{11}	......	y^1_{1f}	......	y^1_{1F}
	2	y^2_{11}	......	y^2_{1f}	......	y^2_{1F}
	......	......	......	......	......	......
	......	......	......	......	......	......
	......	......	......	......	......	......
	n_1	y^{n1}_{11}	......	y^{n1}_{1f}	......	y^{n1}_{1F}
......	......	......	......	......	......	......
......	......	......	......	......	......	......
h	1	y^1_{h1}	......	y^1_{hf}	......	y^1_{hF}
	2	y^2_{h1}	......	y^2_{hf}	......	y^2_{hF}
	......	......	......	......	......	......
	......	......	......	......	......	......
	n_h	y^{nh}_{h1}		y^{nh}_{hf}		y^{nh}_{hF}
......	......	......	......	......	......	......
......	......	......	......	......	......	......
H	1	y^1_{H1}	......	y^1_{Hf}	......	y^1_{HF}
	2	y^2_{H1}	......	y^2_{Hf}	......	y^2_{HF}
	......	......	......	......	......	......
	......	......	......	......	......	......
	n_H	y^{nH}_{H1}	......	y^{nH}_{Hf}	......	y^{nH}_{HF}

The matrix $\mathbf{T}_{3,2}$ can be considered as the product of two matrices:

$$\mathbf{T}_{3,2} = \mathbf{S} \times \mathbf{Y} \qquad [1]$$

This breakdown allows us to understand how the personal income distribution is influenced both by the factorial distribution between capital and labour (matrix **Y**) and by the distribution of individual/household human capital and wealth ownership/endowments (matrix **S**).

The matrix **Y** (Table 3) shows the distribution of value added to different factors. This process is enlightened in the matrix $\mathbf{T_{2,1}}$ of the SAM (Table 1). The macroeconomic variables, which cannot be controlled by a single individual and/or by the households, determinate the distribution of the added value, depending on the use of different technologies and by its changes.

TABLE 3. – *Matrix Y*

$$\mathbf{Y} = \left| \begin{array}{cccccc} \mathbf{Y_1} & \mathbf{0} & \cdots\cdots & \cdots\cdots & \cdots\cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{Y_2} & \cdots\cdots & \cdots\cdots & \cdots\cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots\cdots & \mathbf{Y}_f & \cdots\cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots\cdots & \cdots\cdots & \cdots\cdots & \mathbf{Y}_F \end{array} \right|$$

- Each element s_h^f of the matrix **S** represents the share (y_h^f / Y^f) of the *h-th* group of households to the *f-th* type of income (wages, salaries, income from autonomous work,…) according to the different ownership of human and physical capital.

TABLE 4. - *Matrix* ***S***

$$
\mathbf{S} = \begin{vmatrix}
s_1^1 & s_1^2 & \ldots.. & s_1^f & \ldots.. & s_1^F \\
s_2^1 & s_2^2 & \ldots.. & s_2^f & \ldots.. & s_2^F \\
\ldots. & \ldots. & \ldots.. & \ldots. & \ldots.. & \ldots.. \\
s_h^1 & s_h^2 & \ldots.. & s_h^f & \ldots.. & s_h^F \\
\ldots. & \ldots. & \ldots.. & \ldots. & \ldots.. & \ldots.. \\
s_H^1 & s_H^2 & \ldots.. & s_H^f & \ldots.. & s_H^F
\end{vmatrix}
$$

The SAM as a simulation model: the M "Accounting Multipliers"

The ***multiplier approach based on a SAM*** allows us to estimating the impact on the households' income distribution, and on the whole system, of an exogenous "injections" of income in some accounts.
The new equilibrium can be obtained as a solution of the SAM once it is considered as a linear model, following the same process as in the ***input-output analysis***.
The equilibrium solution ***implies that*** the ***endogenous accounts*** (***Activities***, ***Factors, Private Institutions: Household*** and **Companies**) can be isolated from the ***exogenous ones*** obtained by aggregating some accounts of the original SAM (i.e. ***Government***, ***Rest of the World***, ***Capital/Saving***).

Table 5 - The ***Aggregate SAM***

	Endogenous Accounts				
	Activities	Factors	Private Institutions	Exogenous Institutions	Total
Activities	$\mathbf{S_{11}}$ *(nxn)*	**0**	$\mathbf{S_{13}}$ *(nxh)*	$\mathbf{x_1}$ *(nx1)*	$\mathbf{t_1}$ *(nx1)*
Factors	$\mathbf{S_{21}}$ *(mxn)*	**0**	**0**	$\mathbf{x_2}$ *(mx1)*	$\mathbf{t_2}$ *(mx1)*
Private Institutions	**0**	$\mathbf{S_{32}}$ *(hxm)*	$\mathbf{S_{33}}$ *(hxh)*	$\mathbf{x_3}$ *(hx1)*	$\mathbf{t_3}$ *(hx1)*
Exogenous Institutions	$\mathbf{l'_1}$ *(1xn)*	$\mathbf{l'_2}$ *(1xm)*	$\mathbf{l'_3}$ *(1xh)*	$\mathbf{x_4}$ *(1xn)*	$\mathbf{t_4}$ (1x1)
Total	$\mathbf{t'_1}$ *(1xn)*	$\mathbf{t'_2}$ *(1xm)*	$\mathbf{t'_3}$ *(1xh)*	$\mathbf{t'_4}$ *(1xn)*	

- We can obtain the matrices of expenditure A_{jk} dividing each element in the transaction matrices of endogenous accounts S_{jk} by the $\hat{\mathbf{t}}$ diagonal matrix whose elements are the components of the transposed vector $t'_{k.}$.

$$\mathbf{A_{jk}} = \mathbf{S_{jk}}\left(\hat{\mathbf{t}}_{\mathbf{k}}\right)^{-1} \qquad [2]$$

The normalisation of the transaction matrices S_{jk} allows that the constraints relating to row and column totals of the SAM in Table 1 to be rewritten isolating the group of the *r* (three in our case) endogenous accounts from the exogenous ones.

Starting from the equilibrium relations in Table 5

We can, thus, write

$$t_{end} = A\, t_{end} + x_{end} \qquad [3]$$

$$t_4 = ll'_1 t_1 + ll'_2 t_2 + ll'_3 t_3 + x_4 \qquad [4]$$

where $\mathbf{ll'_k} = \mathbf{l'_k}(\hat{\mathbf{t}}_{\mathbf{k}})^{-1}$

Equation [4] indicates that the equilibrium values of the accounts relating to exogenous Institutions is achieved once endogenous accounts are in equilibrium. Finally, considering the previous equations and the accounting principle that total receipts must equal total outlays, it follows that, in aggregate, total injections into the system must equal total leakages

The equilibrium conditions expressed in equation [4] allow that only equation [3] can be taken into consideration and that it can be rewritten, obtaining the multipliers matrix **M,** as

$$\mathbf{t}_{\mathbf{end}} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{x}_{\mathbf{end}} = \mathbf{M}\, \mathbf{x}_{\mathbf{end}} \qquad [5]$$

$$\mathbf{M} = (\mathbf{I} - \mathbf{A})^{-1} \qquad [6]$$

The formulation in equation [5] indicates that the vector $\mathbf{t}_{\mathbf{end}}$ of the total receipts for each endogenous account can be obtained from the vector $\mathbf{x}_{\mathbf{end}}$, expressing the total receipts of exogenous Institutions multiplied by $(\mathbf{I} - \mathbf{A})^{-1}$, that is by the generalised inverse **A**.

The matrix **M** is referred as the accounting multiplier matrix and shows the overall effects, direct, indirect due to the income generation process closed-loop, from the exogenous injection (originated by a change in one or more components of the exogenous demand) to the endogenous accounts.

The accounting multipliers matrix **M** (Table 6) can be interpreted as a simplified model of the actual way the system is working. More precisely the results of the multiplier analysis can be interpreted as a demonstration of how the economic system is expected to behaves in case the model assumptions perfectly reflect the real situation

Table 6 – Multiplier Matrix M

	Endogenous Accounts			
	Activities	Factors	Private Institutions	Total
Activities	M_{11}	M_{12}	M_{13}	t_1
Factors	M_{21}	M_{22}	M_{23}	t_2
Private Institutions	M_{31}	M_{32}	M_{33}	t_3
Total	t'_1	t'_2	t'_3	

The multiplier matrix **M** assumes a precise meaning with reference to a structural analysis of the income distribution of the Institution Households. The elements of the matrix **M** related to this Institution have the meaning, at a disaggregated level, of a Keynesian expenditure multiplier. Its value depends on the linkages built in the SAM (consumption expenditure, input-output relationships, value added distributed to different household groups according to their ownership of the production factors).

Focusing our attention on the determination of the income distributed within the endogenous Private Institutions the corresponding $\mathbf{t_3}$ vector is given by:

$$\mathbf{t_3} = \mathbf{M_{31}}\,\mathbf{x_1} + \mathbf{M_{32}}\,\mathbf{x_2} + \mathbf{M_{33}}\,\mathbf{x_3} \qquad [7]$$

The column of row totals of the matrices $\mathbf{M_{31}}$ $\mathbf{M_{32}}$ and $\mathbf{M_{33}}$ (table 6) represents really “income” multipliers. Each value indicates by how much the overall income of the corresponding Private Institution (in particular of the row Households decile) would rise if the incomes of each column account (Activities, Factors or Private Institutions) is exogenously increasing by one unit. The row of column totals, instead, indicates the multiplier effect on the income of all Private Institutions when the income of the corresponding column (Activities, Factors, Private Institutions) is exogenously increased by one unit.

The determination of “accounting multipliers” matrix M for the Italian economic system.

The multipliers matrix **M** for the Italian economic system has been obtained starting from a previous work in which a Sam for Italy, has been obtained for the year 1984, after processing data drawn from different sources and after introducing many simplifying hypothesis.

The matrix $\mathbf{S_{32}}$ **(r**esulting from the aggregation of T_{32} and T_{42} of the SAM in table1) shows the ownership of factors by each Private Institution (Households and Companies).The transactions values (Table 6) are “gross” or market incomes. The matrix $\mathbf{S_{32}}$ results from the product of the two matrices **S**, **Y** of the SAM for Italy 1984.

The endogenous account are: Activities, Factors and Endogenous Private Institutions (Households and Companies). Households have been classified in 10 groups (deciles of population) according to their level of disposable (net) income.

All the other accounts of the SAM were aggregated into the vector of Exogenous Institutions.

We focus on matrix $\mathbf{S}_{32}$ (Table 7) that shows the income earned by each Private Institution from each factor. The shares of factors income of each decile are quite different. From the 5th to the 8th decile the share of income from employees income (over total income) is higher than 70%. In the first four deciles, instead, it is lower, and still lower in the last two deciles, accounting for only 64.5% and 52.8% respectively.

The income shares from *Self-employment* and from *Capital in productive activities* assume the highest values in the last decile, even if for self-employment the range of values are lower (from 9,3% to 16,6%), while for productive capital the range is from 4,7% to 18,5%.

The shares of total income of each decile are a signal of the degree of inequality of earnings from factors ownership. Overall, the income of the last decile weights almost a quarter of the total while the first two deciles together get only the 4.9%.

TABLE 7 - Matrix $\mathbf{S_{32}} = \mathbf{S} \times \mathbf{Y}$ (SAM for Italy 1984). Ownership of factors by Private Institutions. Transaction values in millions of lire*

Private Institutions	Employees Income	Self-Empl. Income	Product Capital	Housing Capital	Financial Capital	Totals
1^ decile	5286	883	390	1224	483	8266
2^ decile	14089	2706	1481	1644	766	20686
3^ decile	18936	3433	2284	2060	1045	27758
4^ decile	23365	4121	2958	2156	1269	33869
5^ decile	30126	3781	2647	2348	1695	40597
6^ decile	32619	5102	3500	2904	1712	45837
7^ decile	39218	5583	3531	2802	2366	53500
8^ decile	46706	7187	6027	3638	2875	66433
9^ decile	53719	11620	8836	4434	4664	83273
10^ decile	75595	23748	26491	7688	9776	143298
Companies	0	0	60096	4254	0	64350
Totals	339659	68164	118241	35152	26651	587867

*The matrix is bordered with the row and column of totals

A first numerical examples allows us to determinate the global multipliers matrix **M,** and in particular $\mathbf{M_{32}}$ and $\mathbf{M_{33}}$ quantifying how much a unit exogenous injection in the income of one or more endogenous accounts translates into different amounts of disposable income for all Private Institutions. The matrix **M** can be considered as a "structural measure" of the personal income distribution.

Its value allows us to argue that: i) in the Italian economy, the level of inequality in the Private Institutions', and in particular in the Households income distribution, seems to be a structural feature of the system. ii) the final inequality level is different from the starting one depending on the endogenous accounts that benefits first from the exogenous injection; iii) an exogenous increase in the disposable income of the first two deciles and, even more of the first four, benefits all households deciles and in particular the richest ones.

Matrix $\mathbf{M_{32}}$ (Table 8) shows the values of the accounting multipliers for any unit exogenous injection of income $\mathbf{x_2}$ (transferred from the RoW) to the different factors. $\mathbf{M_{32}}$ reflects the links between functional and personal income distribution, by quantifying how any injection/change in the factors income composition ends up in a differentiated change of the income of the Private Institutions, and in particular in the Households income distribution.

Table 8, Matrix $\mathbf{M_{32}}$ (SAM for Italy 1984) of Private Institutions' Income Multiplier for a unit exogenous injection $\mathbf{x_2}$ that is, for an exogenous injections of income to different factors

Private Institutions	FACTORS					Totals	Averages
	Employees Income	Self-Empl. Income	Product Capital	Housing Capital	Finacial Capital		
1^ decile	0,02588	0,02291	0,00953	0,04095	0,02799	0,12726	0,02545
2^ decile	0,06663	0,06395	0,0276	0,06452	0,05276	0,27546	0,05509
3^ decile	0,08941	0,08283	0,03948	0,08265	0,07136	0,36573	0,07315
4^ decile	0,10961	0,09982	0,0495	0,09123	0,08661	0,43677	0,08735
5^ decile	0,13635	0,10142	0,05163	0,10219	0,10912	0,50071	0,10014
6^ decile	0,15078	0,12764	0,0628	0,12276	0,11653	0,58051	0,11610
7^ decile	0,17844	0,14263	0,06865	0,12716	0,14892	0,66580	0,13316
8^ decile	0,21685	0,18194	0,09932	0,16293	0,18365	0,84469	0,16894
9^ decile	0,25968	0,26836	0,13599	0,20254	0,27195	1,13852	0,22770
10^ decile	0,40552	0,52479	0,33157	0,35644	0,54152	2,15984	0,43197
Total Househ.	1,63915	1,61629	0,87606	1,35337	1,61041	7,09528	1,41905
Companies	0,14542	0,14084	0,58297	0,23164	0,13967	1,24054	0,24811
TOTAL	5,16826	5,05428	3,68289	4,55152	5,02480	23,48174	4,69635

*The matrix is bordered with column of totals and rows of total

A reading by row shows the effects on each decile's disposable income due to an exogenous increase of one unit directed toward the Factor in column (*2,15984= 17 times 0,12726).*
Column totals shows the effects produced on the households as a whole by one unit injection to the column's Factor. Employed labour does not seem to play a dominant multiplying effect as compared to Self-employment and Financial capital, though its value is the higher one: 1,63915 compared to 1,61629 and to 1,61041.

The rows totals of $\mathbf{M}_{33}$ (Table 9) reflects the inequality in the income distribution of the Private Institutions. The rows totals, for the deciles of the Household sector, show a monotonically upward trend. The value for the first decile (1,10911) is rather small and it indicates the reduced potential of the system to translate an exogenous injection $\mathbf{x}_3$ (value transfers from the Government and the RoW to the private institutions) in primary income for the poorest. On the opposite, the multiplier effect in favour of the last decile (2,98571) appears to be particularly strong.

The column totals indicate the income generating power of each decile toward the households as the whole. The multipliers values show that the first four deciles, and particularly the second and the third, have the highest multiplying power. Diagonal elements of matrix $\mathbf{M}_{33}$ are income multipliers within each Private Institution (deciles and Companies) generated by an additional unit of primary income exogenously attributed to the group itself. With reference to the Households deciles, they are all higher than one and they show a monotonically growing trend from the first to the last decile. For an equal exogenous injection of additional income, the final effect within the poorest groups is always weaker than within the richest ones. The poorest deciles, furthermore, have a lower ability to generate income for themselves than for all the households.

TABLE 9. - Matrix $\mathbf{M}_{33}$ (SAM for Italy 1984) of Private Institutions' Income Multiplier for a unit exogenous injection $\mathbf{x}_3$ (transfers from the Government and the RoW to the private institutions).

Private Institutions	*1^decile*	*2^decile*	*3^ decile*	*4^ decile*	*5^decile*	*6^ decile*	*7^ decile*	*8^decile*	*9^decile*	*10^ decile*	*Total Househ.*	*Companies*	*Total*	*Averages*
1^ decile	1,01157	0,01216	0,01217	0,01158	0,01040	0,01117	0,01115	0,01068	0,01007	0,00815	1,10911	0,00297	1,11208	0,10110
2^ decile	0,02810	1,02962	0,02970	0,02835	0,02544	0,02734	0,02721	0,02596	0,02447	0,01983	1,26601	0,00705	1,27305	0,11573
3^ decile	0,03759	0,03964	1,03974	0,03795	0,03406	0,03661	0,03643	0,03474	0,03275	0,02654	1,35606	0,00940	1,36545	0,12413
4^ decile	0,04554	0,04805	0,04820	1,04605	0,04134	0,04442	0,04420	0,04211	0,03970	0,03218	1,43180	0,01147	1,44328	0,13121
5^ decile	0,05304	0,05601	0,05620	0,05374	1,04825	0,05186	0,05163	0,04916	0,04635	0,03758	1,50381	0,01467	1,51848	0,13804
6^ decile	0,06104	0,06441	0,06461	0,06175	0,05543	1,05957	0,05929	0,05648	0,05325	0,04316	1,57899	0,01609	1,59508	0,14501
7^ decile	0,07008	0,07403	0,07428	0,07105	0,06379	0,06856	1,06823	0,06495	0,06124	0,04965	1,66585	0,01968	1,68553	0,15323
8^ decile	0,08835	0,09329	0,09361	0,08952	0,08037	0,08637	0,08595	1,08183	0,07715	0,06254	1,83897	0,02382	1,86279	0,16934
9^ decile	0,11316	0,11948	0,11986	0,11458	0,10286	0,11054	0,10992	0,10469	1,09870	0,07999	2,07379	0,02932	2,10311	0,19119
10^ decile	0,20420	0,21550	0,21615	0,20660	0,18543	0,19928	0,19798	0,18866	0,17784	1,14408	2,93571	0,04743	2,98314	0,27119
Total Househ.	1,71267	1,75220	1,75451	1,72117	1,64736	1,69571	1,69200	1,65926	1,62152	1,50369	16,76009	0,18189	16,94198	1,54018
Companies	0,14610	0,16351	0,16559	0,16240	0,14911	0,15810	0,15583	0,15007	0,14241	0,11863	1,51176	1,01608	2,52784	0,22980
TOTAL.	4,51555	4,72266	4,74677	4,58288	4,21797	4,45819	4,43780	4,25710	4,07318	3,49550	43,50761	1,46120	44,96881	4,08807

A second numerical example can help to better catch the links between the functional and the personal income distribution. A new modified matrix **S** (Table 10) has been obtained introducing the hypothesis that factors' ownership by the household groups is different from the original one.

In particular: 1)the total employees income remains the same; 2) the shares of this factor owned by the richest deciles become higher while those of the poorest become smaller. The SAM' accounting constraints requires the invariance of the rows and of columns totals. Therefore, also, the shares of the other factors owned by each decile have been changed. This implies that the total inequality remains unchanged. A comparison of the "original" multiplier matrix $\mathbf{M_{32}}$ (Table 8) with the modified $\mathbf{M_{32}}$ (Table 11) shows some slights differences. All the multipliers values referred to the Households in the column of Totals show a monotonically upward trend. However, for the first seven deciles these values are all lower. In the presence of a greater concentration of employees' income, there is a lower capacity of all factors to generate income for the poorest.

Both numerical examples enlighten the links between functional and personal income distribution. They show a reduced potential of the exogenous injections to the factors to generate income for the poorest, while the multiplier effect in favour of the last decile appears to be stronger when the distribution of employment income is more unequal.

TABLE 10. – Modified Matrix **S** (SAM for Italy 1984). Factors' ownership shares by each decile of Households and by Companies.

Private institutions	*Employees income*	*Self-empl. income*	*Product capital*	*Housing capital*	*Finalcial capital*
1^ decile	0,00589	0,03496	0,00753	0,05189	0,04386
2^ decile	0,02355	0,05437	0,02944	0,10366	0,06960
3^ decile	0,03533	0,07237	0,03200	0,11550	0,11185
4^ decile	0,05299	0,08466	0,03897	0,07982	0,10071
5^ decile	0,08869	0,05547	0,02239	0,06680	0,06360
6^ decile	0,09603	0,07485	0,02960	0,08261	0,06424
7^ decile	0,11546	0,08191	0,02986	0,07971	0,08878
8^ decile	0,14718	0,08196	0,04632	0,08642	0,08776
9^ decile	0,17858	0,14626	0,05781	0,06924	0,12675
10^ decile	0,25628	0,31319	0,19782	0,14332	0,24284
Companies	0,00000	0,00000	0,50825	0,12102	0,00000

TABLE 11.- Modified Matrix $\mathbf{M_{32}}$ of Endogenous Institutions' Income Multiplier for a unit exogenous injection $\mathbf{x_2}$.

	Factors						
	Employees Income	*Self-Empl. Iincome*	*Product Capital*	*Housing Capital*	*Financial Capital*	*Totals*	*Averages*
1^ decile	0,01773	0,04683	0,01481	0,05865	0,05607	0,19409	0,03882
2^ decile	0,05177	0,08263	0,04672	0,12124	0,09867	0,40103	0,08021
3^ decile	0,07192	0,10902	0,05449	0,13971	0,14955	0,52468	0,10494
4^ decile	0,09567	0,12740	0,06536	0,11219	0,14468	0,54530	0,10906
5^ decile	0,13566	0,10249	0,05234	0,10414	0,11196	0,50659	0,10132
6^ decile	0,14999	0,12887	0,06362	0,12501	0,11981	0,58730	0,11746
7^ decile	0,17753	0,14404	0,06959	0,12974	0,15269	0,67359	0,13472
8^ decile	0,22361	0,15847	0,09491	0,14882	0,16646	0,79228	0,15846
9^ decile	0,27482	0,24262	0,11857	0,15070	0,22587	1,01257	0,20251
10^ decile	0,43119	0,48833	0,30524	0,28949	0,42304	1,93729	0,38746
Total Housh.	1,62989	1,63070	0,88564	1,37967	1,64882	7,17472	1,43494
Companies	0,14381	0,14320	0,58468	0,23619	0,14650	1,25439	0,25088
TOTAL	5,12213	5,12606	3,73065	4,68250	5,21623	23,8777	4,77551

Summing up:

1) in the industrialized countries personal income distribution is inextricably tied up with the functional distribution of income.
2) the deciles' multiplying power changes when the factors' ownership changes.
3) different sources of inequality may come from the distribution of the endowments and of the individual/households "entitlements" (matrix **S**) but also from the institutional and productive structures (matrix **Y**).
4) the traditional redistributive and fiscal policies, aimed at redistributing income and wealth between people, reduce inequalities in the distribution of disposable incomes *ex-post*. They cannot go to the root of the problem.
5) innovative and structural policies (ex-ante), directed toward the *ex-ante* income generating process aimed at changing the composition of the ownership of endowments by different group of households (matrix **S**), are needed.